IJITCE

ISSN 2347- 3657

International Journal of Information Technology & Computer Engineering

Email : ijitce.editor@gmail.com or editor@ijitce.com

International Journal of Information Technology & Computer Engineering
ISSN 2347–3657
Volume 15, Issue 3, 2024

A Review on Mathematics for Engineers: Fundamentals and Applications

Shiva Adithya Velagapudi[1] | Usha Hasini Velagapudi[2] | Mohammed Mukthar Hussain[3]

[1]FIITJEE Junior College, Vijayawada, Andhra Pradesh, India
[2]Delhi Public School, Vijayawada, Andhra Pradesh, India
[3]Professor, Dept. of BSH, Nimra College of Engineering and Technology, Nimra Nagar, Kothur, IBM, Andhra Pradesh, India

To Cite this Article

Shiva Adithya Velagapudi 1, Usha Hasini Velagapudi 2, Mohammed Mukthar Hussain 3, "A Review on Mathematics for Engineers: Fundamentals and Applications" International journal of Information Technology and Computer Engineering, Volume 15, Issue-3 Sep-2024, pp766-777

ABSTRACT

This review article addresses the topic of mathematics for engineers, considering its fundamentals and applications. Mathematics is framed as the language of engineering. The educational objectives of the document consist of clarifying some concepts and using simple examples, problems, and solutions. At the end of the review, the issues addressed are summarized, and some additional considerations on this matter are made. Mathematics is an abstract science that involves the study of numbers, quantities, structures, and shapes and their relationships in different contexts. A possible division of mathematics can be into pure and applied mathematics. Pure mathematics concerns mathematical conceptions and relationships without any relation to real life or practical problems, whereas applied mathematics refers to math that has a practical goal, i.e., applying mathematics to problems of nature and engineering. It can be said that science and technology are based on mathematics; therefore, nothing that involves knowledge or creates new knowledge can be dissociated from math. In this way, engineering is mathematics, as engineering is the creation and fabrication of artifacts by converting an idea (an abstraction) into matter (a tangible); general mathematical models (equations), specific parameters (problems), mathematical computer simulations (calculations), and experiments (testing).

1. INTRODUCTION

Mathematics has existed for centuries, from arithmetic through advanced topics such as calculus, algebra, differential equations, linear algebra, and probability and statistics. It has been extensively used in every aspect of life. Mathematics has also been regarded as the universal language in many disciplines: science, engineering, business, information technology, medicine, and sociology, with the development of special mathematical techniques for applications in particular disciplines: classical mechanics, solid and fluid mechanics, heat transfer, computer graphics, signal processing, control engineering, etc. Mathematics plays a vital role in all engineering disciplines, namely mechanical engineering, civil engineering, electrical engineering, electronics engineering, chemical engineering, and biomedical engineering. However, mathematics is a major barrier for some engineering and engineering technology undergraduates, especially those from unprivileged backgrounds. There is a vision that all engineering undergraduates have a strong mathematical background, are quantitatively trained, and have a deep understanding of engineering mathematics. With this effort, more engineering and technology experts would be educated and contribute to improving local communities. The advancement of engineering mathematics education, especially for engineering undergraduates, is desired and justifiable. This knowledge gallery will be useful and fruitful to mathematics and engineering scholars, society, and the community as a whole. Physics has the mathematical paradigm: start with knowledge, pose questions, form mathematics, predict, and check experiments. In this paradigm, engineering is less explored. Engineering should improve with mathematics. The mathematical paradigm for engineering could be a similar understanding as physics. Based on the understanding and modeling of mathematical engineering science, concerned mathematics becomes active variables. Mathematics is addressed rigorously with active variables to predict and check engineering, forming a hierarchy of mathematical engineering science, namely, engineering under parameters mathematics, engineering under mathematical functions, engineering science, mathematics under engineering, science under mathematics, and mathematics under notation, which are constructed. The hierarchy is mathematically explored. The improvement of engineering with mathematics has been illustrated by diverse ordinal engineering activities [1-2].

2. MATHEMATICS FUNDAMENTALS

Mathematics is a wide-ranging topic that consists of many individual sub-disciplines. Algebra, geometry, differential equations, statistics, and probability are the more common areas that most children learn in school. Calculus, linear algebra, and differential equations are more focused and advanced skills that are increasingly important for engineers to understand. More detailed and in-depth discussions of each individual area are beyond the scope of this review. However, a brief overview of each discipline is provided here to familiarize individuals with both basic terminologies as well as applications relevant for engineers [3].

Calculus is most simply described as the mathematics of change. This can be visualized in terms of graphs as a slope of a curve and in terms of equations as a rate of change of a variable with respect to another variable. The basic principle in calculus is the limit. From limits come derivatives and integrals, which are the two fundamental operations in calculus. Describing a broadened view of intuition in calculus, derivatives gauge the rate of change for time-invariant variables and integrals sum or accumulate the effect of instantaneous contributions in space-variant variables [4].

A function is the fundamental concept in calculus. A function can be defined as a relation between variables, usually between independent and dependent variables. It relates dependent variable(s) to independent variable(s). Independent variable(s) can take any value in the defined range, but the dependent variable(s) will vary according to the rules set by the function. There are many types of functions, such as polynomial functions, rational functions, trigonometric functions, logarithmic functions, exponential functions, etc.

A function that relates a single dependent variable 'y' to a single independent variable 'x' is termed a single variable function. For a single variable function, calculus operations can be performed easily and include differentiation and integration. The basic mathematical notations to define a single variable function are provided below:

Let 'f' be a single variable function, then

$y = f(x)$

when $x \in R$; x is the 'x' variable, y is the dependent 'f' function.

Linear algebra allows engineers to analyze large systems of equations that govern many problems in a more efficient way than traditional methods. Linear algebra is most commonly concerned with computations involving matrices and vectors – arrays of numbers represented in either two or one chain respectively. Matrices can be enlarged or shrunk only by appending or eliminating rows and columns respectively. A special type of matrix is called a square matrix in which the number of rows is equal to the number of columns. Each element of a square matrix can be denoted by an index that refers to the row and column numbers separating it from the other elements in the matrix [5].

Differential equations allow engineers to quantify the change in physical systems as a function of time. Often, mathematicians separate their study of differential equations into ordinary and partial domains. Simply put, an ordinary differential equation has only a single independent variable while partial differential equations consider multiple independent variables describing how a variable changes with respect to both time and space [6].

Derivative: The derivative of a function is defined as the rate of change of this function with respect to its independent variable. In other words, the change in the output of the function with respect to the change in the input of the function is called derivation. The derivative of the 'f' function at point 'a' may be defined as;

$$f'(a) = \lim(h \to 0)\ [f(a + h) - f(a)]/h$$

Meaning that for small values of 'h', how much the 'f' function varies when the 'x' value is varied by 'h' points.

2.1. CALCULUS

Calculus is a branch of mathematics that models and analyzes change, often referred to as "the mathematics of change." Calculus has two principal branches: differential calculus and integral calculus, which are connected by the fundamental theorem of calculus. Both can be applied to numerical or stochastic problems; although for simple problems, mathematical analysis often provides a more practical alternative. As a rule of thumb, calculus is generally appropriate when a modeling problem is expressed in terms of ordinary or partial differential equations, systems of such equations, or stochastic differential equations. It may apply in other situations as well, for instance, when optics is modeled using the wave equation [7].

Differential calculus is the study of the rates at which quantities change. Given a function describing a quantity that depends on another variable such as time, the derivative of this function indicates how the quantity varies with time. More formally, the derivative of a function at a point is the limit of the ratio of the difference between the function evaluated at (a + h) and a to h as h tends toward zero. This limit describes how the function varies at the point. Continuous functions can be differentiated on intervals, with the derivative being another continuous function whose graphical representation describes how the first function varies. The second function can then be differentiated again, yielding the second derivative, which indicates the speed of the change of the first derivative [8].

Iterating the differentiation process gives rise to the concept of higher-order derivatives. In this context, calculus is sometimes referred to as "the mathematics of change." In this way, a wide variety of problems, including mechanical, kinetic, thermal, electrical, economic, biological, and even social, environmental, and political problems, can be analyzed mathematically in terms of ordinary differential equations or partial differential equations, depending on the number of independent variables. With the parametric evolution of physical systems, either deterministic or stochastic, the study of time-dependent quantities such as position, shape, velocity, acceleration, mass, or electric charge naturally leads to mathematical models that are described in terms of differential equations [9].

2.2. LINEAR ALGEBRA

The area of mathematics to be discussed here is linear algebra. It is a branch of mathematics concerning linear equations, linear functions, and their representations through matrices and vector spaces. Linear algebra is central to almost all areas of mathematics and has applications in engineering, physics, computer science, economics, and social sciences. Engineers often use linear algebra in performing tasks such as fitting curves to data, querying data in databases, modeling economic systems, and analyzing structures subject to external forces and designing controllers for automatically operating devices and process systems. Familiarity and comfort level with mathematics topics and concepts are required for engineering students to study complex engineering topics such as system dynamics, control systems, engineering vibrations, advanced circuit analysis, finite element methods, and modern control. Topics and concepts discussed are linear equations, vectors, vector spaces and subspaces, linear combinations and dependence, bases and dimension, rank and basis of row and column space, row echelon and reduced row echelon forms of a matrix, operations with matrices, and linear transformations between vector spaces. Other topics discussed are one-to-one, onto, invertibility, and algebra of matrix transformations; systems of linear equations and their solutions; existence and uniqueness of solutions; homogeneous systems of linear equations; non-homogeneous systems of linear equations; Gaussian elimination; Gauss-Jordan elimination; applications in curve fitting; Eigen values and eigenvectors; diagonalization of a square matrix; and applications in engineering dynamics, models of parameters to be estimated, and vibrations of multi-degree freedom systems [10].

2.3. DIFFERENTIAL EQUATIONS

Differential equations describe the essential characteristics of processes in numerous physical systems encountered in engineering. With the advent of computers and advances in numerical methods, the solution of differential equations has gained ample ground. A differential equation is an equation that relates a function to its derivatives. A function can be a temperature, velocity, concentration distribution, etc., depending on the process. The function of the process at a given instant is mostly required to solve the problem. Therefore, a differential equation is defined as an equation involving an unknown function of a variable (or variables) and its derivatives with respect to that variable. The equation with respect to a single independent variable is called an ordinary differential equation, and a partial differential equation involves two or more independent variables [11].

Differential equations with respect to one independent variable, which may be real or complex, and the function with its derivatives may be polynomial, algebraic, or transcendental, and the derivatives may be finite or infinite, are classified into different types. The ordinary differential equation is classified as linear or non-linear based on the linearity of the function and its derivatives. A linear ordinary differential equation, whether dependent or independent variable, is a linear combination of the function and all its derivatives of the first degree with constant or non-constant coefficients, whereas a non-linear ordinary differential equation involves terms greater than the first degree or higher-order derivatives. In a linear differential equation, if the highest derivative is of order n, then the equation is classified as first, second, third, and so on in the case of the non-linear [12-13].

Engineering problems are mostly governed by the laws of physics. As a result, partial or ordinary higher-order differential equations arise from mathematical modeling. However, the exact analytical solution is difficult or impossible to find for a majority of differential equations. Therefore, alternative approaches are warranted to arrive at an approximate solution. The foundation of a successful approximation and numerical method is laid by the proper understanding of accurate concepts in calculus, algebra, and differential equations [14].

This topic deals with the explanation of the basic concept of differential equations along with the methods of finding analytical and approximate numerical solutions of ordinary and partial differential equations. Special reference is made to the methods used and revised in the numerical analysis packages.

3. APPLICATIONS IN ENGINEERING

An overview of engineering applications for mathematics is presented here. Mathematics has many applications in the engineering field. Branches of mathematics like algebra, geometry, and calculus, which are used daily, are analyzed here. Applications of algebra in various engineering fields are discussed along with some classic equations. Applications of geometry and calculus in science and engineering, specifically in the mechanical engineering field, are discussed. Mathematics is one of the most important subjects in all aspects of life. Mathematics has vast applications in each and every field. Mathematics is very essential in the engineering field, or mathematics is the mother of all engineering fields. There are various branches of mathematics; some of them are algebra, geometry, calculus, trigonometry, and statistics. All branches of mathematics are very essential. There are many engineering applications for each and every branch of mathematics. An overview of engineering applications for mathematics is presented here. Algebra is used in many engineering fields like computer engineering, electrical engineering, and electronics engineering. In computer engineering, computers and microprocessors use algebraic calculations. In electrical engineering, classic equations use algebraic calculations. In electronics engineering, concepts of electronics use algebra. Algebra has great importance in engineering. Algebraic equations are used in engineering applications like the generation of power, voltages, and analysis of electric circuits. Algebra is part of mathematics used for solving problems. It contains equations, one or more variables, and arithmetic operations, along with relation symbols. Geometry is also a branch of mathematics that is widely used in the engineering field. Geometry is concerned with different shapes and their characteristics. Geometry is an important aspect of civil engineering, mechanical engineering, automobile engineering, and architecture. In civil engineering, geometry is needed to construct bridges and buildings. The shape of pillars and the elevation of buildings consider the concepts of geometry. In mechanical engineering, cogwheels and gear cases are designed considering the basic shapes of cylinders, cones, and prisms. Geometry is important in automobile engineering. Geometry is applicable in many engineering fields. Applied science involves concepts of geometry. Many applications of science, like the design of machines, structures, skyscrapers, flyovers, satellites, and ships, are based on geometry. Calculus is one of the branches of mathematics that is extensively used in science and engineering. It is the mathematical study of continuous change. Calculus is applicable in the fields of mechanical engineering, electrical engineering, and civil engineering. There are many concepts in the fields of science and engineering that can be better understood via calculus, like acceleration, velocity, speed, work done, force, pressure, current, capacitance, field intensity,

ISSN 2347–3657

Volume 15, Issue 3, 2024

and frequency of an oscillating system. Understanding these concepts is very essential for students who are pursuing a career in science or engineering 15.

3.1. MECHANICAL ENGINEERING

Engineering is a broad field, and many domains, such as mechanical engineering, electrical engineering, civil engineering, etc., have been created to explore further in any particular area. This review focuses on mechanical engineering, its fundamentals, and important applications in a variety of fields. The world has been vastly affected by the advancements in science and technology. All the developments and operations of machinery in many industries are controlled by different design programs. These programs require knowledge of mathematics. Mathematics is the extensive field of studying numbers, measurements, shapes, and arrangements. It is a discipline required for many professions, primarily for engineers. Mathematics has huge significance in engineering and plays a crucial role in the design process, especially in mechanical engineering disciplines. Mechanical engineering is the discipline of engineering that makes use of mechanics and mathematics in designing and manufacturing machinery. There are many applications of mathematics that are frequently used in mechanical engineering, such as trigonometric functions, calculus, matrices, limits, vectors, etc. Each branch of mathematics holds a different perspective in the field of engineering applications. For a broad vector field used in mechanical engineering applications, Hamilton's principle is considered. Hamilton's principle is the foundation for many application codes in engineering disciplines. It is a general variation principle of mechanics, which is intrinsic to the Lagrange equations of motion. There exists a large collection of such codes for fluid and structural mechanics, elasto-plasticity, heat conduction, wave propagation, etc. All the application programs based on Hamilton's principle are derived from general field equations and variation principles. Hamilton's principle contains both the equations of motion and the boundary conditions governing the motion of a mechanical system. The equation of motion is an important equation used in the field of mathematics for the modeling process of any discipline. The equation used in Hamilton's principle for mechanical engineering applications is described in detail [16].

3.2. ELECTRICAL ENGINEERING

Electrical Engineering is the branch of engineering that deals with the generation, transmission, distribution, and utilization of electrical energy. It generally addresses the problems associated with large-scale electrical systems, as well as smaller electronic systems, like circuit designs. Broadly, the field of Electrical Engineering can be divided into two categories: Power Systems and Electronics Systems, another field where mathematics is heavily utilized.

Electrical energy is generated using various natural resources like coal, hydrocarbons, natural gas, diesel, water, sunlight, nuclear fission, etc. It can also be generated using some bio-resources, like biogas, which are environmentally friendly. Power system engineers utilize mathematical instruments, like differential equations, numerical techniques, and computer algorithms, to design and analyze various power generation stations, distribution and sub transmission networks, power quality issues, earth fault protection of networks, relay coordination, etc., thereby ensuring that the electrical energy generated by these plants can be efficiently transmitted at a low cost and with minimum losses to the end users, large establishments, and general consumers. Mathematical modeling is also done to evaluate fault analysis, like load flow study, to prepare for unexpected problems in the power system [17].

All the electrical energy generated is utilized at various industrial and consumer establishments to run machines, industries, computers, and electrically charging batteries, etc. Techniques like Fourier series, Laplace transformations, and Z-transformations are largely used to design various electrical appliances, especially electric machines: DC machines, induction motors, synchronous motors, converters, transformers, etc. These electrical devices cause electrical and mechanical vibrations in the systems, which need to be reduced for long life and trouble-free operation in the machines and equipment [18].

Computer systems and electronics are now an inevitable part of modern life; right from airport reservation systems to washing machines, TVs to computers, everything involves the use of mathematics. From telephone communication to satellite communication, everything is based on the theory of electronics. Mathematics is utilized from the design of circuits to the manufacture and assembly of printed circuit boards, operational amplifiers, and multi-vibrators.

3.3. CIVIL ENGINEERING

Mathematics plays a vital role in the civil engineering field and its applications. This paper aims to review how mathematics is used in the design, construction, and maintenance of civil engineering infrastructure. The review commences on various mathematical techniques adopted in the civil engineering discipline.

The civil engineering field encompasses four primary areas: geo-techniques, structural engineering, transportation engineering, and hydrology-hydraulic engineering. The sophistication of civil engineering mathematical models has grown throughout the last century. At the present time, computer-aided design and computer-aided engineering systems do not recognize the significant civil engineering mathematical models. Besides, mathematical models for problems in civil engineering are often slight.

Geotechnical engineering is divided into a few tasks: examination of soil samples, testing the thermal properties of soil, and controlling soil reinforcement. For the first two tasks, various chemical and physical methods are in use. Models such as drag and radial basis function models are presented for the fourth task of modeling large-portion coefficient-of-friction tests. Four different modeling strategies are explored: no pre-filtering, moving-window polynomial regression, a conventional ANN, and a fuzzy reasoning-based ANN. Statistical analysis is employed on predictions to elaborate on the importance of non-linear projection techniques [19].

Reinforced concrete structures are designed according to the performance assessment method against seismic issues by a verified set of non-linear seismic analysis procedures and numerical modeling approaches. The five performance levels contemplated in the design are evaluated in terms of probabilistic mathematical models of intensity–damage type for the Rayleigh distribution spectral compatibility MDTs with soil trade nuclei and plants' post-earthquake scenario [20].

Global, cyclic, and risk-based failure models for assessing the used capacity of earth cofferdams to sustain draught-induced stress combinations are briefly presented. Nonetheless, empirical–statistical machine-learning techniques are in focus, such as artificial neural networks, self-organizing maps, support vector machines, and Gaussian processes, as they have proved their power in describing complex and fast-decaying or variable responses. Some of the suggested techniques have been previously adapted to civil engineering mathematical modeling tasks and prove to be an excellent benchmark and basis for enhancement of the knowledge transfer among academic experts [21].

4. CONCLUSION

This review highlights the importance of mathematics for engineers and provides a comprehensive overview of mathematical techniques applied to solve engineering problems. It emphasizes the need for engineering mathematics to develop mathematical models of physical systems, derive solutions, and analyze them for further applications. Advanced mathematical skills and numerical techniques are necessary to tackle complex engineering problems where traditional methods are no longer efficient. The application of calculus, probability, fuzzy logic, statistics, numerical techniques, finite difference method, finite element method, and other models find relevance in electrical, electronics, mechanical, metallurgical, civil engineering, and bioengineering fields. In particular, mathematical modelling of systems governed by partial differential equations is a significant task for process engineers, and it discusses finite difference method and finite element method modeling and simulation of various engineering processes. In addition, a new mathematical technique, the modified decomposition method, can be used to solve nonlinear engineering problems, including atmospheric pollutant dispersion in a confined space, and is simple and powerful. This review can serve as a valuable resource for students and engineers to access diverse engineering problems and the mathematics applied to solve them. However, engineering mathematics should be viewed as a toolbox or workshop that needs to be skilled in, rather than as a rigid mathematic framework.

References:

1. Artigue M. Didactic engineering in mathematics education. Encyclopedia of mathematics education. 2020. [HTML]
2. Usmonov M. General Concept of Mathematics and Its History. INDEXING. 2024. academicsbook.com
3. Noonburg VW. Differential equations: from calculus to dynamical systems. 2020. 43.143.80.206
4. Frank K, Thompson PW. School students' preparation for calculus in the United States. ZDM–Mathematics Education. 2021. pat-thompson.net
5. Farin G, Hansford D. Practical linear algebra: a geometry toolbox. 2021. farinhansford.com
6. Gray WG, Leijnse A, Kolar RL, Blain CA. Mathematical tools for changing scale in the analysis of physical systems. 2020. [HTML]
7. Ely R. Teaching calculus with infinitesimals and differentials. ZDM–Mathematics Education. 2021. [HTML]
8. Hughes-Hallett D, Gleason AM, McCallum WG. Calculus: Single and multivariable. 2020. [HTML]

9. Mkhatshwa TP. Calculus students' quantitative reasoning in the context of solving related rates of change problems. Mathematical Thinking and Learning. 2020. [HTML]
10. Nicholson WK. Linear algebra with applications. 2020. calstate.edu
11. Garabedian PR. Partial differential equations. 2023. [HTML]
12. Parand K, Aghaei AA, Kiani S, Zadeh TI, Khosravi Z. A neural network approach for solving nonlinear differential equations of Lane–Emden type. Engineering with Computers. 2024 Apr; 40(2):953-69. [HTML]
13. Ahmad H, Khan TA, Stanimirovic PS, Shatanawi W, Botmart T. New approach on conventional solutions to nonlinear partial differential equations describing physical phenomena. Results in Physics. 2022 Oct 1; 41:105936. sciencedirect.com
14. Garabedian PR. Partial differential equations. 2023. [HTML]
15. Alam A. Challenges and possibilities in teaching and learning of calculus: A case study of India. Journal for the Education of Gifted Young Scientists. 2020. dergipark.org.tr
16. Abrahamson D, Nathan MJ, Williams-Pierce C, Walkington C, Ottmar ER, Soto H, Alibali MW. The future of embodied design for mathematics teaching and learning. In Frontiers in Education 2020 Aug 25 (Vol. 5, p. 147). Frontiers Media SA. frontiersin.org
17. Entezari A, Bahari M, Aslani A, Ghahremani… S. Systematic analysis and multi-objective optimization of integrated power generation cycle for a thermal power plant using Genetic algorithm. Energy Conversion and …. 2021. [HTML]
18. Weideman JAC, Fornberg B. Fully numerical Laplace transform methods. Numerical Algorithms. 2023. colorado.edu
19. Martinez A, DeJong J, Akin I, Aleali A, Arson C, Atkinson J, Bandini P, Baser T, Borela R, Boulanger R, Burrall M. Bio-inspired geotechnical engineering: principles, current work, opportunities and challenges. Géotechnique. 2022 Aug; 72(8):687-705. icevirtuallibrary.com
20. Cantagallo C, Terrenzi M, Spacone E, Camata G. Effects of multi-directional seismic input on non-linear static analysis of existing reinforced concrete structures. Buildings. 2023. mdpi.com
21. Fu X. Statistical machine learning model for capacitor planning considering uncertainties in photovoltaic power. Protection and Control of Modern Power Systems. 2022. ieee.org