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Complete lift of Affine Connection to the Complex Tangent Bundle

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Abstract

In this paper, complete lift of affine connection on the complex manifold M to its tangent bundle TM are studied.

Introduction:

Several authors have introduced complete lift on the tangent bundle TM of a smooth manifold M using notations of complete lifts on manifold M, but no natural conjecture has been presented for study of complex structure on tangent bundle. This demands introduction of some new construction, which we shall prefer to call the construction of complex analytic tangent bundle of a complex manifold and in brief complex tangent bundle by TM_{2n}.

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2. Complete Lifts of Affine connection on TM_{2n}:

Let ∇ be the covariant differentiation of affine connection on complex manifold M_{2n}, and then there exist a unique affine connection of TM_{2n} whose covariant differentiation ∇^C satisfies

$$\nabla^C_{z^C}(U^C) = (\nabla_z U)^C$$

where $Z, U \in \Im^1_0(M_{2n})$.

Let $\Gamma^\alpha_{\beta\gamma}$ be the connection component for ∇ with respect to local coordinates system $z^1, z^2, \ldots\ldots\ldots, z^n$ with respect to induced co-ordinate system $z^1, z^2, \ldots\ldots\ldots, z^n, v^1, v^2, \ldots\ldots\ldots, v^n$ of TM_{2n} .we set

$$\tilde{\Gamma}^{\alpha}_{\beta\gamma} = \Gamma^\alpha_{\beta\gamma}, \tilde{\Gamma}^{\alpha}_{\beta\tilde{\gamma}} = 0$$

$$\tilde{\Gamma}^{\alpha}_{\tilde{\beta}\gamma} = 0, \tilde{\Gamma}^{\alpha}_{\bar{\beta}\bar{\gamma}} = 0$$

$$\tilde{\Gamma}^{\alpha}_{\beta\gamma} = \frac{\partial \Gamma^\alpha_{\beta\gamma}}{\partial z} v^1, \tilde{\Gamma}^{\tilde{\alpha}}_{\beta\tilde{\gamma}} = \Gamma^\alpha_{\beta\gamma},$$

$$\tilde{\Gamma}^{\tilde{\alpha}}_{\tilde{\beta}\gamma} = \Gamma^\alpha_{\beta\gamma}, \tilde{\Gamma}^{\tilde{\alpha}}_{\tilde{\beta}\tilde{\lambda}} = 0$$

Where the indices with $\sim$ refer to $v^1, v^2, \ldots\ldots\ldots, v^n$, then $\tilde{\Gamma}$'s are connection component of ∇^C.

Proposition 2.1: If T and R are the torsion and curvature tensor fields of affine connection ∇, then T^C and R^C are respectively the torsion and curvature tensor fields of ∇^C.

Proof: proposition follows from the following formulae

$$T^C(Z^C, U^C) = (T(Z,U))^C = (\nabla_{z^U} - \nabla_{U^z} - [Z,U])^C$$

$$= \nabla^C_{z^C} U^C - \nabla^C_{U^C} Z^C - [Z^C, U^C]$$

$$R^C(Z^C, U^C)V^C = (R(Z,U)V)^C([\nabla_z, \nabla_U]^V - \nabla[Z,U]^V)^C$$

$$= [\nabla^C_{z^C}.\nabla^C_{z^C}]V^C - \nabla^C_{[Z^C,U^C]} V^C$$

Proposition 2.2: For any tensor field S and any vector field Z on complex manifold M_{2n} We have

$$(i) \nabla^C_{z^C}(S^C) = (\nabla_z S)^C$$

$$(ii) \nabla^C(S^C) = (\nabla S)^C$$

$$(iii) \nabla^C_{z^C}(S^V) = (\nabla_z S)^V$$

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$$(iv)\nabla^C(S^V)=(\nabla S^V)$$

$$(v)\nabla^C_{z^V}(S^C)=(\nabla_z S)^V$$

$$(vi)\nabla^C_{z^V}(S^V)=(\nabla_z S)^V$$

Proof: As usual the suffices to verify these formulae in the special cases, where $S=f\in\Im^0_0 M_{2n}, S=df\in\Im^0_1 M_{2n} and S=U\in\Im^1_0 M_{2n}$

(i)If $S=f$, then $\nabla^C_{z^C}(f^C)=z^C f^C=L_{z^C}f^C=(L_z f)^C=(\nabla_z f)^C$

If $S=U$, then the formula to be verified is nothing but the definition of ∇^C. If $S=df$ or more generally $S=W\in\Im^0_1 M_{2n}$, then

$$(\nabla^C_{z^V}W^C)(U^C)=\nabla^C_{z^C}(W^C(U^C))-W^C(\nabla^C_{z^C}U^C)$$

$$=\nabla^C_{z^C}(W(U))^C-W^C(\nabla_z U)^C$$

$$=(\nabla_z W(U))^C-W^C((\nabla_z U))^C$$

$$=(\nabla_z W(U))^C=(\nabla_z W)^C(U^C)$$

Hence $(\nabla^C_{z^V}W^C)=(\nabla_z W)^C$

(ii) From (i), we have

$$\xi_{z^C}(\nabla^C S^C)=\nabla^C_{z^C}(S^C)$$

$$=(\nabla_z S)^C$$

$$=(\xi_z\nabla S)^C$$

$$=\xi_{z^C}(\nabla S)^C$$

and hence

$$\nabla^C S^C==(\nabla S)^C$$

(iii) If $S=f$,then ,we have

$$\nabla^C_{z^C}(f^V)=L_{z^C}(f^V)$$

$$=(L_z f)^V$$

$$=(\nabla_z f)^V$$

If $S=V$, then we write $Z=\xi^\alpha\frac{\partial}{\partial z^\alpha}$ and $V=\eta^\alpha\frac{\partial}{\partial z^\alpha}$, then

$$\nabla^C_{z^C}(U^V)=\left(\xi^\alpha\frac{\partial\eta^\alpha}{\partial z^\alpha}+\tilde{\Gamma}^{\tilde{\tilde{\alpha}}}_{\beta\tilde{\gamma}}\xi^\alpha\eta^\gamma\right)\frac{\partial}{\partial v^\alpha}$$

$$=\left(\xi^\alpha\frac{\partial\eta^\alpha}{\partial z^\alpha}+\Gamma^\alpha_{\beta\gamma}\xi^\alpha\eta^\gamma\right)\frac{\partial}{\partial v^\alpha}$$

$$=(\nabla_z U)^V$$

Either by a similar calculation using a local co-ordinate, we get

$$\nabla^C_{z^C}(W^V)=(\nabla_z U)^V$$

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For every $W \in \Im_1^0 M_{2n}$

(iv) The proof is similar that of 2(ii).

(v) This may be proved in the same way as 2(i)

$$\nabla^C_{z^C}(\mathrm{S}^C) = \nabla_{z^V}(\nabla^C \mathrm{S}^C)\xi_{z^V}(\nabla \mathrm{S})^C)$$

$$= (\xi_z \nabla S)^V = (\nabla_z S)^V$$

(vi) The proof is similar that of 2(iii).

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